Modelling and Analysis of a Supply Chain with Supply, Production and Distribution Reliability Considerations

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***Abstract*— This paper explores the effects of Supply, Production and Distribution reliabilities in a three-tier supply chain. The central thrust of the work is to examine and analyse on conceptual and analytical bases the effects of reliabilities of the systemic components of the Supply Chain on the overall cost of the Supply Chain. A linear cost-minimization Mathematical Programming Model of a three-stage supply chain is built in which the supply, production and distribution reliabilities determine supplier, production facility and distributor selections and contributing quantities to supplies, productions and distributions of goods respectively. Test experiments are designed as trade-offs of costs and reliabilities incorporated into the variants of the Mathematical Programming Model to study combinations of effects reliabilities on the cost structure of the Supply Chain.**

***Keywords*—** *Supply Chain, Reliability, Distribution, Production, Supplies, Mathematical Programming*

1. Introduction

A supply chain is a system of organizations, people, technology, activities, information and resources involved in the production and movement of products or services from the pre-production stages through the production and distribution to the eventual consumers of the products or services. Supply chain activities transform natural resources, raw materials and components into a finished product that is delivered to the end customer. The rudimentary supply chain is a straight forward network of activities involved in the chain without backward integration, whereas in sophisticated supply chain systems, used products may re-enter the supply chain at any point where residual value is recyclable. Supply chains link value chains.

A supply chain consists of all parties involved, directly or indirectly, in fulfilling a consumer demand or a costumer request. The supply chain does not only include the suppliers, manufacturers and distributors, but also logistic providers, warehousing professionals, retailers, and customers themselves. The chain is inherently complex in the sense that within each component such as the production stage, the supply chain involves other finer details of organisations and services such as inventory, stock-taking, customer services among others. These functions include, but are not limited to, new product development, marketing, operations, distribution, finance, and customer service.

The competitive contemporary global market, advances in mobile communication, internet and e-delivery services, the ever-broadening logistic strategies and heightened customers' expectation on efficiency of delivery without compromising quality of products and services, have forced business enterprises to focus on and invest in the development of virile supply chains. Research has also advanced, given the sophisticated computing and telecommunication capabilities of this age and has motivated the continuous evolution of the supply chain and of the techniques to manage it effectively, thus aiding business conglomerates to

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execute their Supply Chain in order to remain competitive in the ever-growing global market for products and services.

In a typical supply chain, raw materials are procured and items are produced at one or more factories, shipped to warehouses for intermediate storage, and then shipped to retailers or customers. Consequently, to reduce cost and improve service levels, effective supply chain strategies must take into account the interactions at the various levels in the supply chain. The supply chain, which is also referred to as the logistics network, consists of suppliers, manufacturing centres, warehouses, distribution centres, and retail outlets, as well as raw materials, work-in-process inventory, and finished products that flow between the facilities.

2. Literature Review

Supply Chain modelling and Analysis has largely dwelt on inventory decisions to the exclusion of location and Supply Chain players' issues. In Supply Chain design, modelling and management, the long term issues of facility location is equally important as the short term influence of inventory, player selection and facility issues. Issues such as the virility of the Supply Chain facilities, relative competence of the Supply Chain players including suppliers', facilities' and distributors' reliabilities are often largely omitted from Supply Chain design and modelling. Current trend of literature incorporates location and inventory decisions in integrated supply chain design [1]. Even the subject of integrated location and inventory decisions in Supply Chain has been approached from different angles. From the angle of yield uncertainty, [2] and [3] designed location-inventory Supply Chain without considering disruptions in operations. Jabbarzadeh et al [4] examined an integrated supply chain design problem with multiple distribution centres subject to different sorts of disruptions building on the recent developments of integrated supply decisions in the same model such as those of Cui T, Ouyang Y, Shen Z. J. M. [5]. In highly competitive environments, supply chain disruptions can have a severe if not existential impact on the success of the companies involved. Competitive advantage implies the ability to provide products to customers at any time. Lost sales, decreased market share and large contractual penalties are possible results of disruptions [6]. The inability of a supplier to provide the necessary amount of supply can have a major impact on the profitability of the entire supply chain. Martha and Vratimos [7] and Sheffi [8] give various examples of companies that encountered severe problems when their supply chains were disrupted.

Much as disruptions are critical issues in Supply Chain occasioned by natural and non-natural occurrences such as such natural disasters, strikes, and terrorist attacks, other intricate issues short of disruptions also engender risk and unreliability in the Supply Chain. These include measures of fidelity and virility of Supply Chain players and facilities.

A primary concern in a complex supply chain system is the selection of players (suppliers, distributors, logistic outfits etc) within the chain. Since, not all players within a supply chain system play at optimum efficiency, the onus is on supply chain managers to be armed with rational capabilities and information to select players for optimum performance of the Supply Chain. This brings to fore the necessity to functionalise the level of performance, commitment and efficiency of the various players and entities in the chain in decision-making as to: who to make supplies and in what quantities, what factories produce what and in what quantities; and which distributors receive what goods or services and in what measure? An auspicious method to accomplish this is by evaluating the reliabilities of the various entities from records of performance so as to aid decision-making.

Reliability is defined as the probability that an entity, a product or device performs its intended function without failure under specified conditions for a specified period of time within a time horizon.

Supply chain reliability is defined as the probability of the supply chain meeting mission requirements to provide the required supplies, services or products to the critical transfer points within the system within a time frame. Also, a supply chain is reliable if it performs well when parts of the system fail. In today's world where different uncertainties occur, it is very important to incorporate reliability into the supply chain as it helps to keep the supply chain running effectively should any unforeseen event take place to cause the delay or stop in function of any component of the supply chain. This informs the approach used in this paper on the analysis and design of a typical supply chain.

Supply chain reliability is defined as the probability of the supply chain meeting mission requirements to provide the required supplies to the critical transfer points within the system. Also, a supply chain is reliable if it performs well when parts of the system fail" [9]. In his dissertation, Snyder [9] also presented a work to study models that are robust (i.e., perform well with respect to uncertainties in the data, such as demand) and reliable (i.e., perform well when parts of the system fail).

Qi Lian and Zuo-Jun Max Shen (2007) [2] proposed an integrated model for a three-tiered supply chain network with one supplier, one or more facilities and retailers. The model takes into consideration the unreliable aspects of a supply chain and the properties to the optimal solution of the model were analysed to reveal the impact if supply uncertainty on supply chain design decisions. A general solution algorithm for this model was also proposed.

Chopra et al.[10] studied sourcing strategies when both on-going supply uncertainty (caused by machine reliability and congestion of orders, etc.) and the disruption of supply (caused by low likelihood events such as natural disasters) are considered. By studying two single period models, they concluded that bundling disruption and on-going supply uncertainty into a single measure results in higher inventory than optimal, higher supply chain costs than optimal, and an underutilization of reliable supply sources.

Tomlin [11] studied a single-product setting in which a firm can source from two suppliers, one that is unreliable and another that is reliable but more expensive. Suppliers are capacity constrained, but the reliable supplier may possess volume flexibility. Dada M. et al. [12] considered the problem of a newsvendor that is served by multiple suppliers, where any given supplier is defined to be either perfectly reliable or unreliable. By perfectly reliable they meant a supplier that delivers an amount identically equal to the amount desired, as is the case in the most basic variant of the newsvendor problem. By unreliable, they also meant a supplier that with some probability delivers an amount strictly less than the amount desired.

Markus Bundschuh et al. [13] showed that the traditional approach to strategic supply chain design cannot cope adequately with the risk of supplier failures and surmised that disregarding considerations about reliability and robustness of the supply chain and merely focusing on minimum cost can lead to a high likelihood of supplier failures and to severe supply problems.

The gaps left in all these approaches to the incorporation of reliability in the Supply Chain modelling is that none of these directly address explicit use reliability measures in Supply Chain player selection and determination of inventory quantities taking cognisance of adjudged reliabilities of Supply Chain players. This gap is what this current work fills.

A multi-criteria single objective model of a typical 3-stage, multi-supplier, multi-factory and multi-distributor supply chain network incorporating supplier selection and allocation of supplies, allocation of products and allocation of distributors based on pre-determined reliabilities of the component supply chain players is developed. Attempt is also made to investigate the features of this model with respect to its sensitivity to changes in reliabilities, demand and some functional limitations of in the model. In the proposed model, the delivery logistics and inventory costs are not explicitly modelled. Rather, delivery and inventory costs are presumed to be built into the unit supply, production and distribution costs as appropriate. A snap-shot model of a single period is proposed.

The rest of this paper is organised as follows. In Section 3, the proposed model for the thrust of this work is brought to the fore, exhibiting and deducing the various components of the Mathematical Programming informed. Section 4 discusses and illustrates a conceptual design of reliability relationships among players in the proposed supply chain which serve as input into the model developed in the previous section. In Section 5, a test problem, method of its solution and analyses of solutions are discussed. Finally, concluding remarks are made in Section 6 and possible extensions and future research thrusts are highlighted.

3. The Model

A general model to capture atypical supply chain system on a large-scale is developed. In order to effectively capture this system, a multi-product, multi-factory supply chain system is considered. The objective of our model is to minimize the total cost incurred in the supply chain system bearing incorporating the adjudged reliabilities of the individual suppliers, production facilities and distributors. The following assumptions were made in the development of the model: (1) Demands from all distribution centres are met. (2) No shortfall in meeting demands for components or products (3) All factories produce the same products (4) All factories distribute to the same distribution centres.

3.1 Problem Description

The model developed in this paper presumes a supply chain in which selections and allocation of supply quotas are to be made among a set of suppliers of several components or raw materials of production in a 3-stage supply chain based on predetermined reliability indices of supplies made by different suppliers over time. Furthermore, reliability indices are also proposed in this model to be used in allocating production and distribution quotas to a fixed number of production centres and pre-appointed distributors with known demands. Homogeneous sets of production and distribution centres in which all production centres and distributors are capable of producing or distributing all types of products are presumed.

3.2 Model Notations

Index sets

Let i denote the index set of suppliers, $|i| = I$, j the index set of production centres (factories), $|j| = J$ and d denote the index set of distributors, $|d| = D$. Furthermore, let m denote the index set of materials/components for production. $|m| = M$ and p denote the index set of products, $|p| = P$.

Model Parameter Sets

At the supply end, let $r_{ijm}^{(1)}$ $(C_{ijm}^{(1)})$ denote the reliability coefficient (unit cost of supply) associated with supplier i in supplying material m to factory j, while W_{jm} (w_{pm}) denote base stock at factory j (unit quantity required for product p) of material m and N_{jm} $\left(R_j^{(1)}\right)$ the number of suppliers to supply material m (Supply Sub-System Reliability coefficient) to factory j.

On the production side, let $r_{pj}^{(2)}$ $(C_{pj}^{(2)})$ denote the reliability coefficient (unit cost of production) associated with product p at factory j, and B_{pj} $(R_j^{(2)})$ denote base stock of product p (Production Sub-System reliability coefficient) at factory j.

At the distribution end, let $r_{pjd}^{(3)}$ $(C_{pjd}^{(3)})$ denote the reliability coefficient (unit cost of distribution) associated with distribution of product p, from factory j to distributor d while L_{pd} (D_{pd}) $\left[R_{pd}^{(3)}\right]$ denote base stock (demand) [Distribution Sub-System reliability coefficient] of distributor d, with respect to product p.

3.3 Model Decision Variables

Let $x_{ijm}^{(1)}$ denote the quantity of supplies allocated to supplier i, selected to supply material m to factory j, provided, $F_{ijm} = 1$, where $F_{ijm} = \{0, 1\}$ while $x_{pj}^{(2)}$ the quantity of product p produced at factory j and $x_{pjd}^{(3)}$ denote the quantity of product p to be distributed to distributor d from factory j. In this model, it is assumed that all production and distribution centres will be involved in the supply chain.

3.4 Model Development

Objective

The cost of the Supply Chain is taken to be the sum of the costs of supply, production and distribution in the chain. The model seeks to minimise Supply Chain pseudo-cost (cost penalised proportionate to unreliabilities of Supply Chain player).

Definition 1 (Unreliability Penalty term):

Given that the reliability of a component of the Supply Chain is R_s, then the proportionate penalty on Supply Chain cost for unreliability is $(2 - R_s)$. This is easy to deduce. Since reliability is R_s, the unreliability coefficient is $1 - R_s$ and for an absolutely reliable component, $R_s = 1$, the additional unreliability penalty term for the component cost is $1 + (1 - R_s) = (2 - R_s)$.

Corollary

For a case where reliability is not considered and for rational comparison with the case where reliability is considered, the reliability coefficient for a component is taken as $R_s = 0$ and thus the penalty term for each component is taken as $1 + (1 - 0) = 2$.

Thus, the objective variant for penalising cost for reliability consideration is:

$$Min \sum_{m}^{M}\sum_{i}^{I}\sum_{j}^{J}\sum_{p}^{P}\sum_{d}^{D}\{[(2-R_j^{(1)})C_{ijm}^{(1)}\,F_{ijm}x_{ijm}^{(1)}] + [(2-\ R_j^{(2)})C_{pj}^{(2)}\,x_{pj}^{(2)}] + [(2-\ R_{pd}^{(3)})C_{pjd}^{(3)}\,x_{pjd}^{(3)}]\}$$

Subject to:

Constraints

1) Supply, production and distribution bounds

$x_{pj}^{(2)}\, w_{pm} + W_{jm} - \sum_i x_{ijm} \leq 0$ (Supply tier) (1)

$\sum_d x_{pjd}^{(3)} - x_{pj}^{(2)} - B_{pj} \leq 0$ (Production tier) (2)

$D_{pd} - \sum_j x_{pjd}^{(3)} - L_{pd} \leq 0$(Distribution tier) (3)

2) Constraints based on proportionate distribution of supplies to different suppliers, production to different factories and distribution to different distributors based on the weights of their relative reliabilities.

$$x_{ijm}^{(1)} = r'_{ijm}\,.\sum_i x_{ijm}^{(1)} \quad (4)$$

$$\sum_i F_{ijm} = N_{jm} \quad (5)$$

$$x_{pj}^{(2)} = r'_{pj}\,.\sum_j x_{pj}^{(2)} \quad (6)$$

$$x_{pjd}^{(3)} - r'_{pjd}\,.\sum_d x_{pjd} = 0 \quad (7)$$

Where,

$r'_{ijm} = \frac{r_{ijm}}{\sum_i r_{ijm}}$ is the normalized reliability of supplier 'i' to supply material 'm' to factory 'j' (8)

$r'_{pj} = \frac{r_{pj}}{\sum_j r_{pj}}$ is the normalized reliability of factory 'j' to produce product 'p'. (9)

$r'_{pjd} = \frac{r_{pjd}}{\sum_d r_{pjd}}$ is the normalized reliability of distributor 'd' to distribute product 'p' being supplied to it from factory 'j'. (10)

3) Limited Storage Capabilities

$$\sum_i x_{ijm}^{(1)} - S_{jm} \leq 0 \quad (11)$$

4.0 Reliability Modelling

In this work, as opposed to what obtains in literature, considerations for reliabilities are not limited to the suppliers alone, but are extended to other stages and players in the Supply Chain. This way, the reliabilities of the component sub-systems of the Supply Chain can be accounted for and studied.

4.1 The Reliability concept

In Supply Chain systems, the Reliability attribute addresses the ability to perform tasks as expected and focuses on the predictability of the outcome of a process. Typical metrics for the reliability attribute include: on-time, the right quantity, the right quality. Levels of compliance to such measures as delivery quantity accuracy, Customer Commit Date Achievement time, delivery location accuracy, payment and or shipping documentation accuracies, Orders Delivered damage free conformance, Orders Delivered Defect free Conformance as well as warranty and returns can be used on a practical basis to estimate supply chain player's or component's reliability. Hence, deviations from the metrics of reliability are measures of unreliability of a Supply Chain player(s) or components(s).

If the probability that an equipment or product (by extension, a supply chain component or player) put into service from a reference time 0 will fail before a time T, whose failure function, $f(T)$ is captured as a an exponential function, $f(t) = e^{-t/\lambda}$, where λ, is the mean time between failures, the Reliability of the equipment, product, player or component which is the probability that failure will not occur before a time T is given by,

$$R(t) = \Pr(t \geq T) = \int_T^{\infty} e^{-t/\lambda}\, dt = e^{-T/\lambda} \quad (12)$$

When several components are connected in series, in parallel or in combinations of both, the system reliability for the combination can be calculated in stages, aggregating the reliabilities of the sub-system components at each stage depending on whether the components are in series or in parallel using equations combining until the whole system or network is combined into a super system or network connecting the input to the output (Klimov and Merkuryev [14]; Rausand and Hoyland [15]). Reliabilities of combinations of component equations for calculating reliabilities of homogeneous simple series and parallel network of n components each with reliability, R_i, $i = 1, 2, \ldots, n$ are given respectively as,

$$R_{seiries} = \prod_{i=1}^{n} R_i \quad (13)$$

$$R_{parallel} = 1 - \prod_{i=1}^{n} (1 - R_i) \quad (14)$$

In this work, reliability values are used to select a pre-determined number of suppliers from a set supplying different components/materials (m = 1, 2, ... , M), determine quantities of product p = 1, 2, ... , P to be produced in the chain in a number of factories and determine quantities of each type of products to be made available to various distributors from various factories. Schematic representations of the composition of the system of component reliabilities used in this work are presented in the following three sub-sections.

4.1.1 Supply Reliability:

Composition of Supply Reliabilities

In this model, only a sub-set of the total number of suppliers are assumed to be selectable for supplies of specific material types to specific factories. Selections are assumed to be based on record of performance of a period of time, from which the reliability indices are computed. There are varieties of reliability compositions that can be used for selection of suppliers based on their reliabilities. In this work, it is assumed that all suppliers can supply all types of materials needed and so selections are made for the different materials, products and factories based on records of performance which are used to compute reliabilities composed as in the figure below.

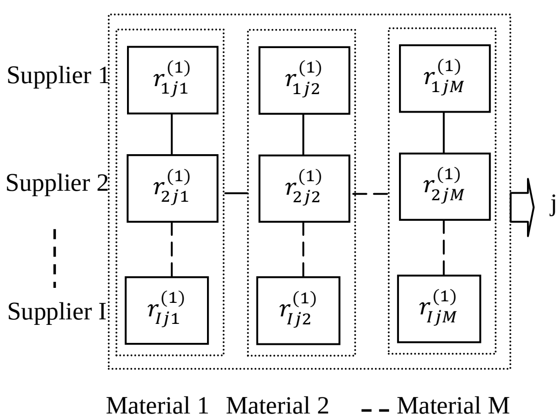

Figure 1: Composition of Supplier Reliabilities

Supply Reliability Aggregation

For each material supply to a factory j, the reliabilities can be aggregated over all suppliers as

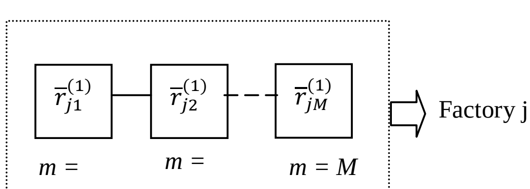

Figure 2: Supply Reliability aggregation over all Suppliers

This aggregation mathematically translates to.

$\overline{r}_{jm}^{(1)} = \prod_{i=1}^{I}\left(1 - r_{ijm}^{(1)}\right)$ for each factory j, $j \in [1, J]$, and material m, $m \in [1, M]$ (15)

Furthermore, the reliability composition can be aggregated as,

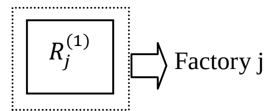

Figure 3: Aggregated Supply Sub-System Reliability

$R_j^{(1)} = \prod_{m=1}^{M}\overline{r}_{jm}^{(1)} = \prod_{m=1}^{M}\left(1 - \prod_{i=1}^{I}\left(1 - \overline{r}_{ijm}^{(1)}\right)\right)$ for each factory j, $j \in [1, J]$ (16)

4.1.2 Production Reliability

Composition of Production Reliabilities

As in the case for supply, production reliabilities are composed for different products based records of performance. Such reliabilities can be computed from knowledge of the state of production facilities and machineries and records previous response to production demand, delivery and customer-satisfaction. In this work, reliability measures of individual factories for the production of different products and distributors are composed and aggregated for use in the supply chain model in order to determine what quantities of different products that the individual factories will produce and for which market or distributor.

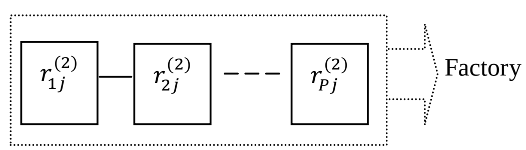

Figure 4: Composition of Production Factory Reliabilities

Production Reliability Aggregation

Production reliabilities are only aggregated over the whole products and for each of the production centres or factories.

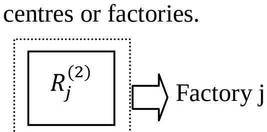

Figure 5: Aggregated Production Sub-System Aggregation

This mathematically translates to the equation,

$R_j^{(2)} = \prod_{p=1}^{P} r_{pj}^{(2)}$ for each factory j, j $\in$ $[1,\ J]$ (17)

4.1.3 Distribution Reliability

Composition of Distribution Reliabilities

Reliabilities are composed for different distributors based records of performance. Such reliabilities can be computed from knowledge of product disposition rate, record of returns, the state of production facilities and customer-satisfaction inquiry among many other factors. In this work, reliability measures of individual distributors for the distribution of different products from different factories are composed and aggregated for use in the supply chain model in order to determine what quantities of different products from individual factories that each distributor will be supplied.

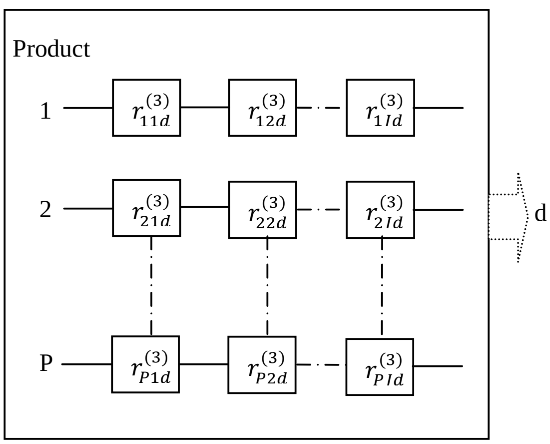

Figure 6: Composition of Distributors' Reliabilities

Distribution reliabilities are only aggregated over the whole production centres or factories and for each of the products and distributors.

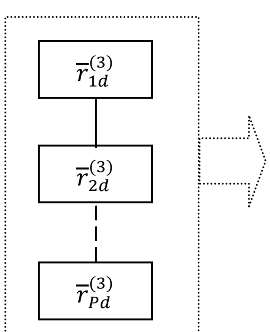

Figure 7: Aggregated Distributor Reliability

It can be expressed as,

$R_{pd}^{(3)} = \prod_{j=1}^{J} r_{pjd}^{(3)}$ for each product p, p $\in$ $[1,\ P]$ and distributor d, d $\in$ $[1,\ D]$ (18)

5.0 Test Problem, Solution Methodology and Analyses

5.1 The Test Problem and Model Parameters

In this work, we considered a supply chain network with 4 suppliers each supplying 3 materials or components to 2 factories or production centres that produce 2 products each. The products are distributed through 4 distribution outlets for which 8 distributors are major. The parameters of the model as enunciated in Section 2.2 are as depicted in Tables 1 – 5 below.

Table 1: Supplier reliability coefficients (unit Costs (× 150 USD) of materials or components)

Value of:		**Value of:**	
$r^{(1)}_{ijm}$ $(c^{(1)}_{ijm})$		$r^{(1)}_{ijm}(c^{(1)}_{ijm})$	
$r^{(1)}_{111}$ $(c^{(1)}_{111})$	0.8(44)	$r^{(1)}_{311}$ $(c^{(1)}_{311}$	0.75(48)
$r^{(1)}_{112}$ $(c^{(1)}_{112})$	0.9(37)	$r^{(1)}_{312}(c^{(1)}_{312}$	0.61(39)
$r^{(1)}_{113}(c^{(1)}_{113})$	0.72(30)	$r^{(1)}_{313}(c^{(1)}_{313}$	0.87(48)
$r^{(1)}_{121}(c^{(1)}_{121})$	0.5(44)	$r^{(1)}_{321}(c^{(1)}_{321}$	0.55(38)
$r^{(1)}_{122}(c^{(1)}_{122})$	0.55(44)	$r^{(1)}_{322}(c^{(1)}_{322}$	0.84(47)
$r^{(1)}_{123}(c^{(1)}_{123})$	0.78(38)	$r^{(1)}_{323}(c^{(1)}_{323}$	0.5(45)
$r^{(1)}_{211}(c^{(1)}_{211})$	0.55(46)	$r^{(1)}_{411}(c^{(1)}_{411}$	0.5(47)
$r^{(1)}_{212}(c^{(1)}_{212})$	0.91(37)	$r^{(1)}_{412}(c^{(1)}_{412}$	0.55(45)
$r^{(1)}_{213}(c^{(1)}_{213})$	0.8(40)	$r^{(1)}_{413}(c^{(1)}_{413}$	0.9(39)
$r^{(1)}_{221}(c^{(1)}_{221})$	0.6(48)	$r^{(1)}_{4212}(c^{(1)}_{421}$	0.82(39)
$r^{(1)}_{222}(c^{(1)}_{222})$	0.61(33)	$r^{(1)}_{422}(c^{(1)}_{422}$	0.56(38)
$r^{(1)}_{223}(c^{(1)}_{223})$	0.85(38)	$r^{(1)}_{423}(c^{(1)}_{423}$	0.7(31)

Table 2: Production reliability coefficients (unit Costs (× 150 USD) of production)

$r_{pj}^{(2)}(c_{pj}^{(2)})$	**Value**
$r_{11}^{(2)}(c_{11}^{(2)})$	0.75(2.5)
$r_{12}^{(2)}(c_{12}^{(2)})$	0.59(3)
$r_{21}^{(2)}(c_{21}^{(2)})$	0.66(4.6)
$r_{22}^{(2)}(c_{22}^{(2)})$	0.89(3.9)

Table 3: Distribution reliability coefficients (unit Costs (× 150 USD) of distribution)

Value of:		**Value of:**	
$r_{pjd}^{(3)}(c_{pjd}^{(3)})$		$r_{pjd}^{(3)}(c_{pjd}^{(3)})$	
$r_{111}^{(3)}(c_{111}^{(3)})$	0.45(0.25)	$r_{211}^{(3)}(c_{211}^{(3)})$	0.55(0.38)
$r_{112}^{(3)}(c_{112}^{(3)})$	0.72(0.6)	$r_{212}^{(3)}(c_{212}^{(3)})$	0.7(0.73)
$r_{113}^{(3)}(c_{113}^{(3)})$	0.77(0.75)	$r_{213}^{(3)}(c_{213}^{(3)})$	0.8(1.13)
$r_{114}^{(3)}(c_{114}^{(3)})$	0.73(0.7)	$r_{214}^{(3)}(c_{214}^{(3)})$	0.6(1.05)
$r_{121}^{(3)}(c_{121}^{(3)})$	0.82(0.5)	$r_{221}^{(3)}(c_{221}^{(3)})$	0.9(0.75)
$r_{122}^{(3)}(c_{122}^{(3)})$	0.47(0.8)	$r_{222}^{(3)}(c_{222}^{(3)})$	0.82(1.2)
$r_{123}^{(3)}(c_{123}^{(3)})$	0.49(0.2)	$r_{223}^{(3)}(c_{223}^{(3)})$	0.56(0.56)
$r_{124}^{(3)}(c_{124}^{(3)})$	0.62(0.45)	$r_{224}^{(3)}(c_{224}^{(3)})$	0.7(0.68)

Table 4: Material and Product Base Stock

Material /Component Base Stock		**Product Base Stock**	
W_{11}	0.05	B_{11}	200
W_{12}	0.08	B_{12}	140
W_{13}	0.06	B_{21}	130
W_{21}	0.09	B_{22}	150
W_{22}	0.05		
W_{23}	0.08		

Table 5: Distribution Base Stock and Product Demand at Distribution Centres

Distribution Base Stock		**Product Demand (Distribution)**	
L_{11}	200	D_{11}	20000
L_{12}	120	D_{12}	10000
L_{13}	150	D_{13}	15000
L_{14}	40	D_{14}	8000
L_{21}	250	D_{21}	15000
L_{22}	150		
L_{23}	100		
L_{24}	50		

5.2 Model Variants and Solution Methodology

Four variants of the Mathematical model were solved for the purpose of comparisons.

Variant 1: The first variant is the case where Supply chain player reliabilities were not considered.

Variant 2: In the second variant the respective suppliers' reliabilities were considered in distribution of supply quotas.

Variant 3: The third variant incorporated both supplier and production reliabilities in the distribution of quotas of supply and production to the various suppliers and production centres or factories respectively.

Variant 4: In the fourth model variant, suppliers', production and distributors' reliabilities were all considered in the distribution of quotas of supplies, production and distributions to the respective players.

The various model variants are solved using the Genetic Algorithm-based Microsoft® Excel Add-In Palisade Evolver 5.7. For the Genetic Algorithm base of Evolver, a constant Crossover rate of 0.65 and mutation rate of 0.1 with roulette wheel selection mechanism were used for all variants of the model.

5.3 Test Problem Solutions and Analyses

The optimal objective values for each of the three tiers of the chain and the sum total are as in Table 6 for each of the four variants of the model.

Table 6: Optimal Objective Costs (× 150 USD) for all Variants of Model

Optimal objective Costs				
Variant	Supply	Product-ion	Distribu-tion	Total Costs
1	2248491.9	671566.6	107677	3027735.4
2	1776530.2	762483.0	128825	2667838.3
3	1644931.0	566223.8	77593.8	2288748.6
4	1648859.8	641795.5	88223.8	2378879.3

Table 7: Reduced Total Costs (× 150 USD)

Variant	2	3	4
Reduced Total Cost	359897.1	738986.8	648856.1

5.3.1 Optimal Cost Comparisons in all variants

Total Costs

The Model variant 1 represents a case in which reliabilities of the supply chain player or sub-system were not taken into consideration. The other variants can therefore be safely compared with it as a measure of savings in cost by taking reliabilities into consideration. A reduction in cost of 359897.1 cost units resulted when the reliabilities of the suppliers were considered and 738986.8 units of cost when the reliabilities of both the suppliers and the production facilities were taken cognisance of. This confirms that production facilities reliabilities also influence the optimal cost to a tune of 379089.8 units of cost, assuming the influence of the supplier reliabilities remain constant. The 648856.1 units of cost reduction when reliabilities at all the three tiers of the chain were considered is still an appreciable reduction in costs compared with the variant 1 case.

Supply Costs

The supply tier cost components depicts the same trend of cost savings as the total cost with reduced unit Costs (× 150 USD) of 471961.7, 603560.9 and 599632.0 for variants 2 to 4 respectively.

Production Costs

Consideration of suppliers' reliabilities in the supply chain alone (variant 1) resulted in marginal increase in the production and distribution cost components although with total cost reductions obviously afforded by the consideration of reliabilities of suppliers. However, in variants 3 and 4 where production facilities' reliabilities were considered cost reductions of 105342.8 and 29771.1 units respectively.

Distribution Costs

The distribution tier cost components also follow the same trend as for the production tier cost components resulting in Costs (× 150 USD) savings of 30083.1 and 19453.0 units for variants 3 and 4 respectively.

5.3.2 Quota Distribution Comparisons across all variants

Supply Quota Distribution

The optimal supply allocation and supplier choices for different products and materials are as in Table 8.

Table 8: Supply allocation to Suppliers for Products and to Factories

Supply Variable	Variants			
	1	2	3	4
$x_{111}^{(1)}$	0.0	0.0	603.7	434.8
$x_{112}^{(1)}$	0.0	5209.1	0.0	0.0
$x_{113}^{(1)}$	3140.8	5830.7	507.3	455.8
$x_{121}^{(1)}$	0.0	0.0	2175.4	1973.6
$x_{122}^{(1)}$	4211.7	5837.1	0.0	0.0
$x_{123}^{(1)}$	1858.8	205.8	3266.8	2992.3
$x_{211}^{(1)}$	0.0	6000.0	4433.0	4657.1
$x_{212}^{(1)}$	0.0	0.0	0.0	0.0
$x_{213}^{(1)}$	3015.2	314.0	0.0	0.0
$x_{221}^{(1)}$	0.0	4310.5	2144.5	1821.2
$x_{222}^{(1)}$	0.0	0.0	0.0	0.0
$x_{223}^{(1)}$	1968.2	299.2	1770.9	1782.3
$x_{311}^{(1)}$	0.0	0.0	0.0	0.0
$x_{312}^{(1)}$	0.0	0.0	3765.2	3954.3
$x_{313}^{(1)}$	0.0	0.0	0.0	0.0

Supply Variable	Variants			
	1	2	3	4
$x_{321}^{(1)}$	1398.0	780.0	220.9	501.1
$x_{322}^{(1)}$	0.0	0.0	3128.0	3377.7
$x_{323}^{(1)}$	0.0	2477.3	0.0	0.0
$x_{411}^{(1)}$	3712.8	0.0	0.0	0.0
$x_{412}^{(1)}$	4925.6	45.4	1246.3	1188.4
$x_{413}^{(1)}$	0.0	0.0	4811.6	5134.1
$x_{421}^{(1)}$	3317.9	0.0	147.8	130.2
$x_{422}^{(1)}$	0.0	0.0	1997.7	1931.7
$x_{423}^{(1)}$	0.0	2628.4	0.0	0.0
Total	27549	33937.6	30219.2	30334.4

The model affords optimal choices of combinations of suppliers for all materials based on reliabilities of the suppliers for optimal costing. From the table the robustness of the model to perform this task is evident. Furthermore, total allocations increased under variants 2, 3, and 4 where reliabilities of the sub-systems are factored into the decision process compared with the case in model variant 1 where reliabilities are not considered. The same trend of increment ensued here as the trend for total optimal costs reduction as explained in the last sub-section.

Production Quota Distribution

Production quota allocation to different production centres or factories for all four variants of the model considered are depicted in Table 9.

Table 9: Production Quota Distribution for all Model Variants

Variant	$x_{11}^{(2)}$	$x_{12}^{(2)}$	$x_{21}^{(2)}$	$x_{22}^{(2)}$	**Total**
1	34694	26590	18267	21859	101410
2	28464	51570	43369	27333	150736
3	47976	36779	16877	22351	123983
4	47976	36779	23379	28192	136326

The Optimal total production increased in model variants 2, 3 and 4 compared with what obtained in model variant 1 following the same trend as the case for optimal costs explained earlier.

Distribution Quota Distribution

Optimal allocations of quantities of products to distributors for all model variants are as in Table 10.

Table 10: Optimal Product allocation to Distributors for all Model Variants

Variable	Variants			
	1	2	3	4
$x_{111}^{(3)}$	12000	500	22790	7198
$x_{112}^{(3)}$	4031	10393	10519	5665
$x_{113}^{(3)}$	12000	500	3273	9488
$x_{114}^{(3)}$	4516	9170	7785	10258
$x_{121}^{(3)}$	8416	23577	4540	11725
$x_{122}^{(3)}$	7276	500	1841	6451
$x_{123}^{(3)}$	2287	18302	11162	7935
$x_{124}^{(3)}$	3047	3481	4383	7785
$x_{211}^{(3)}$	6637	500	13034	7785
$x_{212}^{(3)}$	10404	6449	922	9657
$x_{213}^{(3)}$	500	11933	591	500
$x_{214}^{(3)}$	500	20711	1774	5069
$x_{221}^{(3)}$	7764	12822	1096	5560
$x_{222}^{(3)}$	500	9878	9859	500
$x_{223}^{(3)}$	10238	964	8094	7750
$x_{224}^{(3)}$	2914	500	1857	4304
TOTAL	93030	130180	103520	107630

Just as in the cases of optimal supplier and production allocations depicted in Tables 7 and 8, total optimal allocations increased following the same trend as earlier explained.

6.0 Conclusions

6.1 Extensions

This work has considered factoring reliability measures into cost considerations of a three-tier supply chain model and has amply demonstrated that such considerations can positively influence costs and quantity allocations across all the tiers of the Supply Chain. Several other considerations can be built in. The costs and reliabilities for some were taken as crisp values in this work. They can be specified as fuzzy or stochastic parameters and thus effects of uncertainty ascertained. The model here has also assumed that inventory costs are built into costs for supplies, production and distribution parameters of the model. These can be isolated and integrated into the model. Furthermore, following current trend integrated models of facility location and supply chain component reliabilities can be built.

6.2 Concluding Remarks

In this paper, a multi-stage, multi-Supply Chain-player supply chain model has been developed incorporating the notion of reliabilities of the suppliers, production and distribution in other to minimise network cost. The result has amply demonstrated the necessity of reliability assessment of component players in the Supply Chain network.

References

[1] Shen Z. J., *Integrated Supply Chain Design Models: A Survey and Future Research Directions*, Journal of Industrial and Management Optimization 3: 1-27, 2007.

[2] Qi, Lian; Shen, Zuo-Jun Max, *A Supply Chain Design Model with Unreliable Supply.*, Naval Research Logistics, Vol. 54: 829-844, 2007.

[3] Aryanezhad MB, Jalali SG, Jabbarzadeh A., *An integrated model for designing supply chain network under demand and supply uncertainty*, International Journal of the Physical Sciences, 2011.

[4] Jabbarzadeh. A., Aryanezhad, M. B., Abedi. S., *Designing a Reliable Supply Chain Network with Random Disruption Consideration*, Proceedings of the 41st International Conference on Computers & Industrial Engineering, Los Angeles. 429 – 434, 2011.

[5] Cui T, Ouyang Y, Shen Z.J.M., *Reliable facility location design under the risk of disruptions*, Operations Research 58 (4): 998-1011, 2010.

[6] Bundschuh, M., Klabjan, D., Thurston, D.L., *Modeling robust and reliable supply chains*, Working paper, University of Illinois at Urbana-Champaign, 2006.

[7] Martha, J. and Vratimos, E., *Creating a Just-in-Case Supply Chain for the Inevitable Next Disaster*, Mercer Management Journal, 14, 70-77, 2002.

[8] Sheffi, Y., *Building a Resilient Supply Chain*, Harvard Business Review, 1(8), pp. 1-4, 2005.

[9] Snyder, Lawrence V., *Supply Chain Robustness and Reliability: Models and Algorithms*, A Dissertation submitted to the graduate school in partial fulfilment of the requirements for the degree Doctor of Philosophy in Industrial Engineering and Management Sciences, Northwestern University, 2003.

[10] Chopra, S., U. Mohan, and G. Reinhardt, *The importance of decoupling recurrent and disruption risks in a supply chain*, Working paper, Kellogg School of Management, Northwestern University, 2006.

[11] Tomlin, B. T., *On the value of mitigation and contingency strategies for managing supply-chain disruption risks*, Management Science , vol. 52 No. 5: 639-657, May 2006.

[12] Dada, M., Petruzzi, Nicholas C., Schwarz, Leroy B., *A Newsvendor's Procurement Problem when Suppliers Are Unreliable,* MSOM Winter 2007 vol. 9 No. 1: 9-32, 2007.

[13] Bundschuh, M., Klabjan, D, and Thurston, D. L., *Modeling robust and reliable supply chains*, Optimization Online e-print. www.optimization-online.org , (27-08-2013).

[14] Klimov Ruslan, Merkuryev Yuri, *Simulation Model for Supply Chain Reliability Evaluation*, Baltic Journal on Sustainability, 2008 14(3): 300–311, 2008.

[15] Rausand, M., Hoyland, A., *System Reliability Theory: Models, Statistical Methods and Applications*, John Wiley & Sons Ltd., 2004.

[16] Aryanezhad, M. B., Jalali, S. G., Jabbarzadeh, .A., *An integrated supply chain design model with random disruptions consideration. African Journal of Business Management* 4(12): 2393-2401, 2011.

[17] Aryanezhad, M.B., Jabbarzadeh, A., Tafreshi, P.T., Abedi, S., *An Integrated Supply Chain Design Model under Unreliable Performance of Supplier*, Computers and Industrial Engineering (CIE), 40th International Conference: 1-6, 2010.

[18] Klimov, R., Merkuryev, Y., *Simulation model for supply chain reliability evaluation,* Technological and Economic Development of Economy 14(3): 300–311, 2008.