

# A Model for Forecasting Dependent Demand in Inventory Replenishment Processes

Ismail Civelek<sup>#1</sup>

<sup>#</sup>*Department of Management, Western Kentucky University  
Bowling Green, KY, 42101, USA*

<sup>1</sup>*ismail.civelek@wku.edu*

**Abstract—** This paper studies a capacitated multi-period, multi-product inventory problem under a finite horizon. The main goal is to understand the impacts of evolution of forecasts for demand streams for multiple products on inventory replenishment decisions. This paper provides an input modelling technique to model such complex inventory management problem and offer research questions for future investigations. This paper's main contribution is analysing a complex but very practical problem in inventory management for multiple products and multi-periods when demand streams are dependent of each other and forecasting the demand requires simulation and updating over the finite planning period. This study shows that positively correlated demand has negative effects on the performance of the inventory replenishment processes by low service levels. If the demand is negatively correlated, the impact on the performance of the inventory replenishments is more complex.

**Keywords—** *Inventory Management; Simulation Input Modelling; Supply Chain Management*

## 1. Introduction

This paper considers a capacitated newsvendor problem with multiple products and multiple periods in a finite planning horizon. The demand is considered as a multivariate time series with marginal distributions from the Johnson translation system and an autocorrelation structure specified through finite order of dependence. The objective of the retailer is to minimize total expected costs over time horizon. Rather than assuming an exponentially or normally distributed demand, we use Johnson translation system that provides more general and flexible demand structure. In this study, the retailer does not know the demand distribution exactly; he or she only knows the shape of the distribution that allows him or her to select specific distribution from Johnson family with certain parameters.

In this study, the demand for each period is forecasted via simulation at the beginning of the planning horizon and the forecast is updated after each period; also the demand for the rest of the periods are simulated by updated information at the beginning of every period. After demand is realized as considered to be the new information to the system, the forecast of the rest of the periods and the base stock levels are updated. In terms of updating the forecast of the demand, the retailer changes the parameters of Johnson translation system. Thus, the distribution of the demand is expected to stay within the Johnson family.

This paper investigates very practical, but complex to analyse, research questions. For example, in a manufacturing environment, what would be the safety-stock levels between serial line resources facing correlated interarrival and processing times? Moreover, this study investigates whether the inventory policy change if the manufacturing philosophy is a push system or pull system? The main contribution in this paper is to tackle a very complex but practical problem that has numerous applications in manufacturing, service and telecommunication systems. In Section 2, we survey the related literature and position our work in the operations management literature. Then, in Section 3, the details of the model and main results of this complex problem are studied. Finally, this paper concludes the study in Section 4 and provides discussions of managerial implications and future work in Section 5.

## 2. Literature Review

In management science community, the researchers have been studied the problem of forecast updating, nonstationary and dependent demand structure. Regarding forecast evolution, Ref. [12] considers a model of forecast evolution as a quasi- Markovian or Markovian system and suggests a dynamic programming formulation for the series of conditionally lognormal forecasts. Ref. [13] studies

the capacitated single and multi-product system with terminal demand and no setup costs with lognormal forecasts. They provide a dynamic programming approach and three different heuristics for solving the problem.

Martingale model for forecast evolution (MMFE) is first studied by [10] and [14] independently. Ref. [14] performs a simulation study with a linear programming model for production planning problem. Additionally, Ref. [10] considers multi-period problem and aims to optimize the trade-off between the inventory for a single stage and production capacity. Ref. [11] studies the additive evolution MMFE model in capacitated environment the optimal ordering policy is better in forecast evolution model than the standard inventory model. Ref. [23] considers the MMFE model and stationary demand for single product and capacitated single server under heavy traffic approximations and they characterize the modified base stock policy regarding forecast updating.

Ref. [16] studies the forecast evolution model with bounded forecast bands in a single product, terminal demand and capacitated problem setting and they assume the demand is uniformly distributed within the band. They develop four heuristics and compare them to the optimal solution; their two heuristics give near optimal. Ref. [17] considers the inventory problem where the retailer does not know the demand distribution exactly and they quantify the value of observed demand and the effect of forecasting on the expected costs at the retailer.

Regarding the view of advance demand information in forecast generating problem, Ref. [7] studies when the customers can place orders in advance that provide the firm advanced demand information. They prove that the state dependent ( $s$ ,  $S$ ) and base stock policies are optimal for stochastic inventory problems with and without fixed costs. Ref. [8] studies the problem of optimal inventory policies for multi-echelon inventories under advanced demand information and they show the optimality of state dependent, echelon base stock policies for both finite and infinite time horizon. Furthermore, Ref. [22] studies the similar problem for distribution systems under advanced demand information and he analyses the benefit of advanced demand information on allocation of decisions. Moreover, Ref. [20] studies the impact

of demand updating to supply chain flexibility. They consider two ordering options, one is for functional products and the other one is for fashion innovative products. They use Bayesian demand model for functional products and Martingale evolving demand model for innovative products.

Regarding the simulation literature, Ref. [3] studies the problem of generating multivariate time series input with marginal distributions from Johnson family and an autocorrelation structure in finite order of dependence. In our study, the retailer will specify the parameters of Johnson translation system and the autocorrelation structure; then we generate forecasts for finite planning horizon with specified order of dependence. Similar to [3], Ref. [5] uses advanced simulation input modelling to study of the impact of bivariate and temporal dependencies among interarrival and service times on the performance of single-server queueing systems and reports significant performance decay in queueing systems due to dependence between interarrival and service times. Moreover, Ref. [1] presents a two-level supply chain model consisting of a retailer and a supplier; then it considers the evolving demand as AR (1) process and introduces an adaptive inventory replenishment policy jointly with forecast evolution that utilizes the Kalman-Filter technique. We refer the reader to [2], [4], [6], [9], [21] and [23].

### 3. Model Setting and Results

In simulation modelling, generating input is one of the most crucial problems to represent the uncertainty in the system. For instance, in queueing systems, the interarrival process and service time requirements are mostly considered as independent. If we want to take dependency into account in the system, we have to supply dependent input for simulation.

The model used in this paper provides multivariate dependent input from a wide-variety of marginal distributions. The model uses the rank correlation instead product-moment correlation, because product-moment correlation keeps the linear dependence; but rank correlation keeps also nonlinear relation, so it is more general. The product-moment correlation matrix for a random vector  $X = (X_1, \dots, X_k)$  is the correlation matrix  $\Sigma_x = (\Sigma_{ij} : 1 \leq i, j \leq k)$  where:

$$\Sigma_X(i, j) = \frac{\text{cov}(X_i, X_j)}{\sqrt{\text{Var}(X_i)\text{Var}(X_j)}}$$

However, the rank correlation is in the following form:

$$\Sigma_X(i, j) = \frac{\text{cov}(F_i(X_i), F_j(X_j))}{\sqrt{\text{Var}(F_i(X_i))\text{Var}(F_j(X_j))}}$$

where  $F_i$  and  $F_j$  are the cumulative probability distribution functions of  $X_i$  and  $X_j$ . In a typical  $\text{VAR}_k(p)$  model, we have a  $k$ -variate vector autoregressive process of order of dependence  $p$ . Each variable can be represented as a linear combination of past experiences plus a white noise:

$$Z_t = \alpha_1 Z_{t-1} + \alpha_2 Z_{t-2} + \dots + \alpha_p Z_{t-p} + u_t, \\ t = 0, \pm 1, \pm 2, \dots$$

In this representation,  $Z_t$  is a  $k$  by  $1$  vector and  $\alpha_i$ ,  $i=1, 2, \dots, p$ , is a fixed  $k$  by  $k$  autoregressive coefficient matrix. Moreover, we assume that  $E[U_t] = 0_{(k \times 1)}$  and  $\Sigma_u$  are positive semidefinite.

Regarding the unknown correlation structure of the base process,  $\Sigma_z$ , the model uses the relationship, which is introduced by Ref. [18]:  $\Sigma_z(i, j) = 2 \sin(\pi \Sigma_x(i, j) / 6)$ , where  $\Sigma_x$  is the rank correlation. After finding the autocorrelation matrix of the base process, we get  $\alpha$  from Yule-Walker equations [19]:

$$\alpha = \Sigma \Sigma_Z^{-1}, \alpha = [\alpha_1, \alpha_2, \dots, \alpha_p]_{(k \times k p)} \\ \Sigma = [\Sigma_Z(1), \Sigma_Z(2), \dots, \Sigma_Z(p)]_{(k \times k p)} \\ \Sigma_Z = \begin{bmatrix} \Sigma_Z(0) & \dots & \Sigma_Z(p-1) \\ \vdots & \ddots & \vdots \\ \Sigma_Z'(p-1) & \dots & \Sigma_Z(0) \end{bmatrix}_{(k p \times k p)}$$

The model assumes the stability condition, which means that the roots of the reverse characteristic polynomial lie outside of the unit circle; that is the roots  $|z_i| > 1$  and calculated by:

$$|I_{(k \times k)} - \alpha_1 z - \alpha_2 z^2 - \dots - \alpha_p z^p|.$$

After finding the autoregressive coefficients, the modelling approach can find the white noise vector's,  $u_t$ , correlation matrix by:

$$\Sigma_u = \Sigma_Z(0) - \alpha_1 \Sigma_Z'(1) - \dots - \alpha_p \Sigma_Z'(p).$$

Since the retailer in this problem setting does not know the demand distribution, forecasts are driven by the flexible Johnson family distributions, which are introduced by [15]. In this family of distributions, a random variable  $x$  is defined by the cumulative density function as:

$$F_X(x) = \Phi \left\{ \gamma + \delta f \left( \frac{x - \xi}{\lambda} \right) \right\},$$

where  $\gamma$  and  $\delta$  are shape parameters,  $\xi$  is the location parameter and  $\lambda$  is the scale parameter. The random variable, the demand in our problem setting, is from the following transformations:  $f(y) = \log y$  for the lognormal family;  $f(y) = \log(y + \sqrt{y^2 + 1})$  for the unbounded family;  $f(y) = \log(y/(1-y))$  for the bounded family and  $f(y) = y$  for the normal family. In this problem setting, the retailer specifies the demand distribution by determining  $\gamma$ ,  $\delta$ ,  $\xi$  and  $\lambda$ ; then, her/she makes the initial forecast for the finite planning horizon.

In the generalized model, this paper assumes multiple products,  $i=1, 2, \dots, k$ , finite time horizon,  $t=1, 2, \dots, T$ , and finite order of dependence,  $p=1, 2, \dots, p$ . Moreover, this study assumes full-backlogging, finite inventory capacity; zero fixed ordering cost and zero lead-time. At  $t=0$ , the demand for all periods is forecasted via simulation. Forecasts are adjusted based on new observed data and the demand for the rest of the time horizon is forecasted again via simulation. At the end of the period, the demand is realized and the inventory holding cost and shortage penalty are charged.

At the beginning of the time horizon, the retailer does not know, so a simulation is run to get the demand estimates for each time period.  $X_t$  is the demand process,  $Z_t$  is the base process and  $\rho_x$  and  $\rho_z$  are the corresponding rank-correlations. The optimal policy is a modified base stock policy due to the assumption of zero fixed ordering cost. After demand is realized as considered to be the new information to the system, the forecast of the rest of the periods and the base stock level are updated. In updating the forecast of the demand, we change the parameters of Johnson translation system. Therefore, the distribution of the demand is expected to stay within the Johnson family.

## 4. Discussion

This paper considers a capacitated newsvendor problem with multiple products and multiple periods in a finite planning horizon. The demand is considered as a multivariate time series with marginal distributions from the Johnson translation system and an autocorrelation structure specified through finite order of dependence. The objective of the decision-maker is to minimize total expected costs over finite time horizon. Instead of assuming an exponentially or a normally distributed demand, we use Johnson translation system that provides more general distributions. In this study, the decision-maker does not know the demand distribution exactly. Hence, he or she only knows the shape of the distribution that allows him or her to select specific distribution from Johnson family with certain parameters.

Although this paper does not show the exact optimal policy for inventory replenishment problem with forecast updating in multiple-product and multi-period setup, the consideration of dependence in higher dimension is a promising area of research. Using auto-correlated demand in the context of inventory problems is new and Ref. [1] uses the AR (1) structure in which the order of dependence is one. In this problem setting, the order of dependence is allowed to be more than one. The model in this paper shows that positively correlated demand has negative effects on the performance of the inventory replenishment processes by low service levels. If the demand is negatively correlated, the impact on the performance of the inventory replenishments is more complex.

## 5. Conclusion

This paper leaves several research questions for future work. As an on-going research, the author investigates the safety-stock levels between machines, which are located in a serial production line with correlated processing times. Moreover, extending the current problem into nonzero fixed cost would be a promising area to start. Similarly, assuming realistic, nonzero, assumptions of lead-time would be another extension possibility of this study.

Additionally, the input modelling process used in this paper might fail to generate the correlated

demand in higher dimensions, number of products and order of dependence, due to fact that we need to get positive semidefinite correlation matrix for base process and appropriate autoregressive constant for stability condition. Therefore, as a future research project, the author is also working on mitigating this risk of simulation input-generation failure to investigate an approximation method for forecasting demand with specified correlation by matrix completion methods [9]. Moreover, the author leaves a structural optimal policy analysis for future study.

## References

- [1] Y. Aviv, "Gaining benefits from joint forecasting and replenishment processes: the case of auto-correlated demand", *Manufacturing & Service Operations Management*, Vol. 4, No. 1, pp. 55-74, 2002.
- [2] S. Benjaafar, W. L. Cooper. "Production-inventory systems with imperfect advance demand information and updating", *Naval Research Logistics*, Vol. 58, No. 2, pp. 88-106, 2011.
- [3] B. Biller, B. L. Nelson. "Modeling and generating time-series input processes using a vector autoregressive technique", *ACM Transactions on Modeling and Computer Simulation*, Vo. 13, No. 3, pp. 211-237, 2003.
- [4] I. Civelek. "A commodity production model with operational flexibility of investing optional capacity on offshore platforms", *International Journal of Supply Chain Management*, Vol. 3, No. 3, pp. 1-6, 2014.
- [5] I. Civelek, B. Biller, A. Scheller-Wolf. "The impact of dependence on queueing systems", Working paper, Carnegie Mellon University, Pittsburgh, PA, 2009.
- [6] I. Civelek, I. Karaesmen, A. Scheller-Wolf. "Blood platelet inventory management with protection levels", *European Journal of Operational Research*, Vol. 243, No. 3, pp. 826-838, 2015.
- [7] G. Gallego, Ö. Özer. "Integrating replenishment decisions with advance demand information", *Management Science*, Vol. 47, No. 10, pp. 1344-1360, 2001.
- [8] G. Gallego, Ö. Özer. "Optimal replenishment policies for multiechelon inventory problems under advance demand information", *Manufacturing & Service Operations Management*, Vol. 5, No. 2, pp. 157-175, 2003.
- [9] S. Ghosh, S. G. Henderson. "Behavior of NORTA method for correlated random vector generation as the dimension increases", *ACM Transactions on Modeling and*

- Computer Simulation, Vo. 13, No. 3, pp. 276-294, 2003.
- [10] S. C. Graves, D. B. Kletter, W. B. Hetzel. "A dynamic model for requirements planning with application to supply chain optimization", Operations Research, Vol. 46, No. S3, pp. 35-49, 1998.
- [11] R. Güllü. "On the value of information in dynamic production/inventory problems under forecast evolution", Naval Research Logistics, Vol. 43, No. 2, pp. 289-303, 1998.
- [12] W. H. Hausman. "Sequential decision problems: a model to exploit existing forecasters", Management Science, Vo. 16, No. 2, pp. 93-111, 1969.
- [13] W. H. Hausman, R. Peterson. "Multiproduct scheduling for style goods with limited capacity, forecast revisions and terminal delivery", Management Science, Vo. 18, No. 7, pp. 370-383, 1972.
- [14] D. Heath, P. L. Jackson. "Modeling the evolution of demand forecasts with application to safety stock analysis in production/distribution systems", IIE Transactions, Vol. 26, No. 3, pp. 17-30, 1994.
- [15] N. L. Johnson. "Bivariate distributions based on simple translation systems", Biometrika, Vol. 36, No. 3/4, pp. 297-304, 1949.
- [16] P. Kaminsky, J. M. Swaminathan. "Utilizing forecast band refinement for capacitated production planning", Manufacturing & Service Operations Management, Vol. 3, No. 1, pp. 68-81, 2001.
- [17] H-K. Kim, J. K. Ryan. "The cost impact of using simple forecasting techniques in a supply chain", Naval Research Logistics, Vol. 50, No. 5, pp. 388-411, 2003.
- [18] W. H. Kruskal. "Ordinal measures of association", Journal of the American Statistical Association, Vol. 53, No. 284, pp. 814-861, 1958.
- [19] H. Lütkepohl. "Introduction to multiple time series analysis", Springer-Verlag, 1993.
- [20] J. M. Milner, P. Kouvelis. "On the complementary value of accurate demand information and production and supplier flexibility", Manufacturing & Service Operations Management, Vol. 4, No. 2, pp. 99-113, 2002.
- [21] A. Norouzi, R. Uzsoy. "Modeling the evolution of dependency between demands, with application to inventory planning", IIE Transactions, Vol. 46, No. 1, pp. 55-66, 2014.
- [22] Ö. Özer. "Replenishment strategies for distribution systems under advance demand information", Management Science, Vol. 49, No. 3, pp. 255-272, 2003.
- [23] B. L. Toktay, L. M. Wein. "Analysis of a forecasting-production-inventory system with stationary demand", Management Science, Vol. 47, No. 9, pp. 1268-1281, 2001.